

" ∞ -categories" \Leftrightarrow "quasicategories"

Motivation:

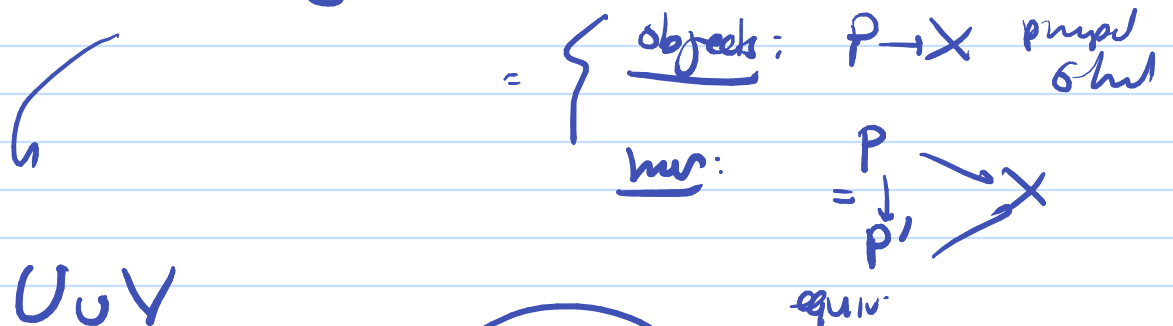
- Higher groupoids. (Grothendieck)

• Sets $X(\mathbb{R}) = \{ (x, y, z) \in \mathbb{R}^3 \mid x^3 + y^3 = z^3 \}$

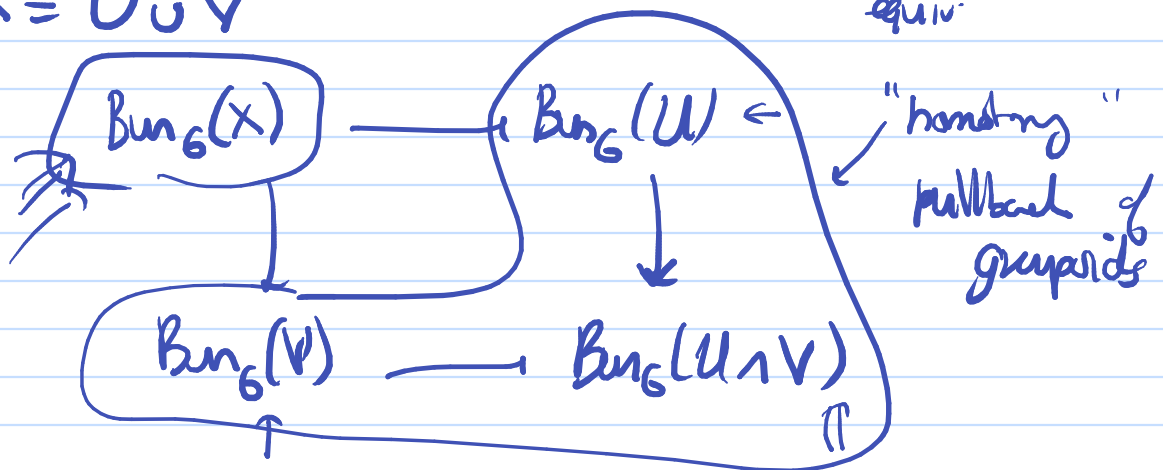
• Ex: $X = \text{trip' space}$, $G = \text{group}$

$\Rightarrow \{ \text{principal } G\text{-bun's on } X \} / \sim = ??$

$\hookrightarrow \text{Bun}_G(X) = \text{groupoid (1-groupoid)}$



$X = U \cup V$



$\text{Bun}_G(X) / \sim = H^1(X, G)$

$$\Rightarrow n\text{-groupoids} \Leftrightarrow H^n(X, A)$$

Ex: $\{1\text{-groupoids}\} \in 2\text{-groupoid}$

↳ objects = groupoids

morphisms = functors

2-morphism = nat. trans of functors

Grothendieck: $\{\infty\text{-groupoids}\} \Leftrightarrow \left\{ \begin{array}{l} \text{homotopy} \\ \text{types of} \\ \text{spaces} \end{array} \right\}$

"Homotopy Hypothesis" \cup

$$\{1\text{-groupoids}\} \Leftrightarrow \{1\text{-trunc. spaces}\}$$

classifying space

$$G \xrightarrow{\quad} BG$$

$e \xrightarrow{\pi_1}$

Combinatorial homotopy theory = simplicial sets \cup

$$\{ \text{homotopy types} \} \Leftrightarrow \{ \text{Kan complexes} \}$$

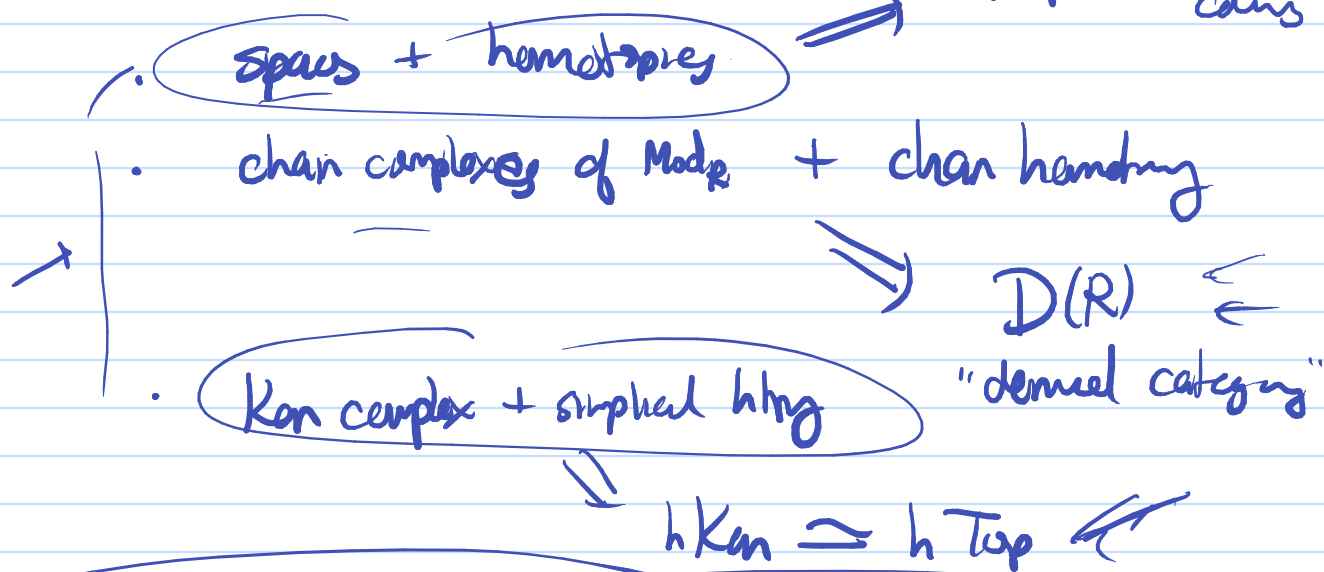
\Downarrow

$\{\infty\text{-groupoids}\}$

\uparrow

}

abstract homotopy theory:



Quillen: Model category \Rightarrow homotopy category

↳ homotopy limits + colimits

Ex:
$$\begin{array}{ccc} U \cup V & \rightarrow & Y \\ \downarrow & & \downarrow \\ U & \rightarrow & X = U \cup V \end{array}$$

↳ Mayer-Vietoris seq in H_p

no point of space

$$F = \begin{array}{ccc} E' & \rightarrow & E \\ \downarrow & \searrow p, b & \downarrow p \\ B' & \rightarrow & B \end{array}$$

(fiber bundle)

↳ lies in \mathcal{T}_{top}

no pullback of spaces

↳ Homotopy limits/colimits are "derived limits" of ord limits/colimits

→ Model category \Rightarrow ∞-category

Span, ch ops, ∞ -categories
 Specht, ..., ;
 Cat $_{\infty}$

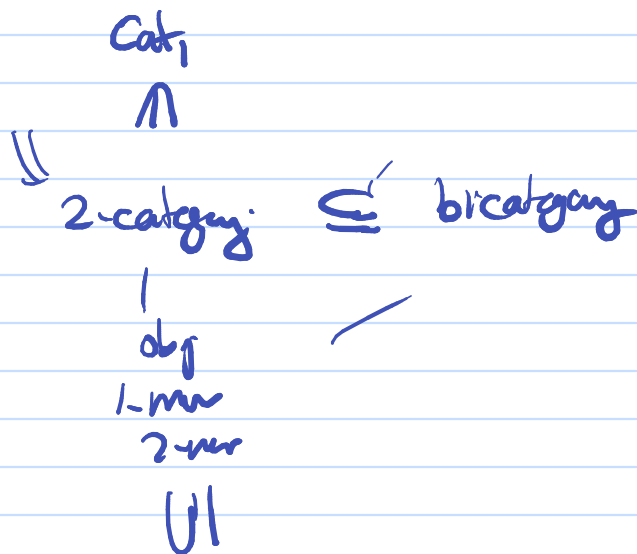
What is an ∞ -category? = quasicategory

Bourgin-Vogt 1972 ("weak Kan complexes")

A. Joyal ~ 2003 (∞ -groupoids (= quasicats st all nps are 1503))
 " Kan complex)

J. Lurie ~ 2006

{ ~ 900 pages (Higher Topos Theory)
 - 1200 pages (Higher Algebra)



∞ groupoid \approx 2 groupoid (all 1-mor, 2-mor invertible)

$\infty \text{ cat} = (\infty, 1) \text{-cateng.}$

quasicats

1-cateng

- objects
- 1-mr
- 2-mr
- 3-mr
- 4-mr

invertible

Δ
 $(\infty, 2) \text{-cats}$

2-cateng

- object
- 1-mr
- 2-mr
- 3-mr
- 4-mr

invertible

dygms
 $\infty \text{ cats}$
 n - m

$\text{Cat}_{(\infty, 1)} = \text{Cat}_2 \in (\infty, 1) \text{-cateng}$

$\text{Cat}_{(6, 1)}$

$(\infty, 2) \text{-cateng}$

- simplicial sets - nerve
- define quasicats -

